Aircraft Dynamically Similar Model Design using Simulated Annealing

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Abstract. A new methodology has been proposed to design a dynamically similar/scaled model (DSM) of aircraft. This method uses the simulated annealing (SA) optimization algorithm to get the maximum similarity between model and full-scale aircraft with help of systems movement and using minimum ballast weight. For the \( \frac{1}{2} \) model of an unmanned aerial vehicle (UAV), internal arrangement is designed to achieve the desired model center of gravity position and moments of inertia. A computer code is developed, and model suitable arrangement is obtained. Results show that the proposed optimization approach to design of DSM was successfully used to find adequate model systems arrangement and minimizing ballast weight to access more capacities for data-acquisition systems or fuels. In this problem, ballast weight reduced about 0.6 kgf for a 55 kgf model, in addition of simplicity of DSM design for various configuration and flight regimes.

Introduction

Different methods in aircraft design and development process are being used for aircraft behavior prediction such as theoretical methods, computational fluid dynamics (CFD) and wind tunnel testing. However, these methods have estimations and errors in comparison with real flight conditions. One of the most powerful techniques for aeronautical evaluation is DSM. DSMs have been widely used during the creation of new flying vehicles, for testing aerodynamic concepts, control systems development and exploring high-risk flight envelopes [1, 2]. DSM is the model which is geometrically similar to the full-scale aircraft and has the similar mass distribution. In conventional DSM design methods, the model must be constructed light enough to get desired mass distribution with the aid of a ballast weight. Finding ballast weight and position methodologies is presented in NASA documents, that are based on using momental ellipsoid equations, which have some limitations such as finding feasible solution, complexity and trial & error [3 – 6]. There are few articles about DSM design, especially via an optimization approach that is focused in this paper. In the Ref. [6], algorithm development for optimum design of aeroelastic DSM is presented. In the present paper, a rigid DSM will be designed using meta-heuristic optimization technique instead of conventional methods to reach flexibility in system arrangement and reducing ballast weight.

Design Problem Definition

The problem is a DSM design for a UAV with a 442 kgf weight. The UAV body length is 5.5m so for simplicity of construction & less Reynolds number differences, scaled ratio is considered \( \frac{1}{2} \). DSM is a powered remotely unmanned vehicle that commercial-of-the-shelf systems are used. Fig. 1 shows the internal arrangement of designing DSM. The arrangement will be modified to satisfy similarity requirements that are presented in Table 1. Full-scale aircraft weight and moments of inertia in one configuration are presented in Table 2. Using Table 1, DSM weight and moments of inertia are calculated and presented in Table 3.

<table>
<thead>
<tr>
<th>Linear dimension</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight, mass</td>
<td>n( \times )( \frac{1}{\sigma} )</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>n( \times )( \frac{1}{\sigma} )</td>
</tr>
</tbody>
</table>

\( n \) is the ratio of model-to-full-scale dimensions, \( \sigma \) is the ratio of air density to that at sea level (\( \rho/\rho_0 \)).

Table 1: The selected required scale factors for dynamic models [1]
Table 2: Full-scale aircraft weights and moments of inertia

<table>
<thead>
<tr>
<th>Weight (kgf)</th>
<th>X_cg (m)</th>
<th>Y_cg (m)</th>
<th>Z_cg (m)</th>
<th>Ixx</th>
<th>Iyy</th>
<th>Izz</th>
<th>Ixz</th>
</tr>
</thead>
<tbody>
<tr>
<td>442.2</td>
<td>3.18</td>
<td>0</td>
<td>0.03</td>
<td>44.8</td>
<td>875.2</td>
<td>19.52</td>
<td>442.2</td>
</tr>
</tbody>
</table>

Table 3: DSM weights and moments of inertia

<table>
<thead>
<tr>
<th>Weight (kgf)</th>
<th>X_cg (m)</th>
<th>Y_cg (m)</th>
<th>Z_cg (m)</th>
<th>Ixx</th>
<th>Iyy</th>
<th>Izz</th>
<th>Ixz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.61</td>
<td>28</td>
<td>27.35</td>
<td>1.4</td>
<td>0.015</td>
<td>0.00</td>
<td>1.59</td>
<td>55.28</td>
</tr>
</tbody>
</table>

At first, for designed initial possible DSM configuration, based on its components weight \( m_i \) and coordinates \( (X_{cg}, Y_{cg}, Z_{cg}) \), an estimation of CG position and moments is obtained. Here for components which have known standard geometrical shape, equations for cube, ball and ellipsoid are used. The uniform mass distribution is considered that does not make noticeable error. After that, using parallel axes theorem, the model’s total moments of inertia are calculated. In Eq. 1, \( I_0 \) is moment of inertia around model CG position and \( d \) is distance of each component to CG position.

\[
I_0 = I + md^2
\]

\[
X_{cg} = \sum_1^n m_i x_i / \sum_1^n m_i
\]

**Design Variables, Objective Function and Constraints**

In this problem, the objective function is the minimization of CG position and moments of inertia values deviation from DSM desired values. Ballast weight and its position and systems position are design variables. Furthermore, systems and ballast allowable locations are constraints that are being implemented in neighborhood creation and changing, that means only possible neighbors can be produced. Another constraint is the model maximum weight that mustn’t exceed the DSM desired value. This problem has several objective functions that must be satisfied all of them. These objective functions are converted to one by summation with weight coefficients \( W_i \). Weight coefficients are estimated then will be corrected after some running.

\[
\text{cost}_f = w_x (I_{xx} - I_{xx_{desired}})^2 + w_y (I_{yy} - I_{yy_{desired}})^2 + w_z (I_{zz} - I_{zz_{desired}})^2 + w_{xz} (I_{xz} - I_{xz_{desired}})^2 + w_{xy} (I_{xy} - I_{xy_{desired}})^2 + w_{yz} (I_{yz} - I_{yz_{desired}})^2
\]

\[
\text{cost}_{f1} = w_{xg} (X_{cg} - X_{cg_{desired}})^2 + w_{yg} (Y_{cg} - Y_{cg_{desired}})^2 + w_{zg} (Z_{cg} - Z_{cg_{desired}})^2.
\]

\[
\text{cost}_{total} = \text{cost}_f + \text{cost}_{f1}
\]
Problem Solving Methodology

This problem is a permutation type because a suitable arrangement of the component is the goal. The SA algorithm has good performance for permutation problems [8]. Moreover, problem characteristics encourage using this method, because some components are in wrong positions that will be disturbed and fixed during problem solving. SA simulates the process of slow cooling of metals to achieve the minimum function value in a minimization problem. In Ref. [8] the SA algorithm is presented. In this algorithm a set of design variables is considered as the start point. Then by design variables disturbing, neighbors are created that if objective function improved, will be accepted else with probability of $e^{-\Delta C/T}$ will be accepted. Here T is the current temperature and $\Delta C$ is the objective function variation. There are several options for mass distribution changing:

1. structure modules weight changing
2. systems position changing
3. using ballast and changing its weight and position

In this problem, 2nd and 3rd methods are used. For a design variable definition, the coordinates of each system are considered as design parameters and by standard methods, DSM CG and moments of inertia are calculated. This is starting point and differs from desired values naturally; therefore, the neighbor solutions are produced and directed by the optimization algorithm. Design variables are disturbed randomly in the positive or negative direction. Coordinates changing have a permissible limitation and program prevents from impossible solution. Moreover, iteration parameter is included in Eq. 6 that cause more change at first and less change in last iterations. The presented methodology is shown in Fig.2.

$$X_i = X_i + \frac{10}{n_{iteration}} \times (\text{rand} - 0.5) \times 0.02 \quad (6)$$

![Algorithm Parameters Adjustment](Fig. 2 Methodology flowchart)

Algorithm Parameters Adjustment. Ref. [8] presents some ways for SA algorithm parameters adjustment. For initial temperature, it offers an estimation that after 100 objective function evaluation, initial temperature is chosen to acceptation rate be 50%, in this problem after several running initial temperature chose equal to 10000.
\[ \tau_0 = e^{-\Delta \xi/T_0} = 50\% \] (7)

Another parameter is markov-chain length, that Ref. [8] proposed 100N for disturbing number that N is the number of design variables. In this problem, 100 iterations are considered in a homogeneous markov-chain that was suitable after several running. Furthermore, various types for cooling function are presented in same Ref. such as linear, geometric and logarithmic. Geometrical model is used here. Lastly, one of the most common criteria for program ending is reaching to final temperature that should be near zero. Final temperature was 0.001 in this problem that showed good results.

**Strategies.** Three strategies are used. At first, one ballast is considered and its weight and position were the design variables. At 2nd strategy two ballasts are used but at 3rd strategy, eight systems coordinate are changed to optimize mass distribution. The goal in 3rd method has been reducing ballast weight.

**Program Results Analysis**

The following figures show computer code sample results that horizontal axes present number of markov-chain for reaching the final temperature. In the figure title, markov-chain length and geometrical cooling rate are written. In the 1st strategy that all components are fixed and only one ballast is considered as design variables, all DSM moments were not converged to desired values. Additionally, CG position has deviation in X direction. After several running, feasible solution was not found. It was as a result of limitation for ballast location within model boundaries and maximum weight of it. Hence, in the 2nd strategy, two ballasts are used. In this case, result convergence to desired value was better than first case, but objective function was not minimized entirely. This is because of maximum weight constraint of the model that means it is impossible to achieve desired mass distribution with limited ballast weight and position. One solution is weight reduction of DSM and increasing ballast that is the common method. Another solution is systems arrangement changing that investigate in 3rd strategy. In this case, two ballasts exist and positions of systems are being changed. It was anticipated that ballast weight reduced relating to other cases. Fig. 3 to Fig. 7 show these case results. It is obvious that the best strategy is using ballast and systems position changing. In the complementary future activities the structure mass distribution can be changed to reach more optimum model mass distribution. Lastly, it is shown in Fig. 3 that objective function were not converged to zero exactly. That is for full-scale aircraft input data that was based on theoretical methods in the design process. The computer optimization code was tested for simple standard shapes, and their DSM data was obtained exactly.

<table>
<thead>
<tr>
<th>Ballast weight</th>
<th>X(m)</th>
<th>Y(m)</th>
<th>Z(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.535 kgf</td>
<td>2.9</td>
<td>0.366</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Table 4: Ballast weight and position in 1st solving method

<table>
<thead>
<tr>
<th>Weight(kg)</th>
<th>X(m)</th>
<th>Y(m)</th>
<th>Z(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballast 1</td>
<td>1.417</td>
<td>-0.018</td>
<td>0.001</td>
</tr>
<tr>
<td>Ballast 2</td>
<td>0.666</td>
<td>1.932</td>
<td>-0.099</td>
</tr>
</tbody>
</table>

Table 5: Ballast weight and position in 2nd solving method
Fig. 3 Objective function reduction in case of 2 ballast and systems position changing

Fig. 4 Ixx convergence to desired value

Fig. 5 Iyy convergence to the desired value

Fig. 6 Izz convergence to the desired value
Conclusion

The most important result from present paper is the possibility of DSM design using the meta-heuristic optimization algorithm such as SA that can be extended for more complex problems. Moreover, systems arrangement changing effect on ballast weight saving that can be replaced by fuel or test equipments.

References


