

Optimal Fuzzy CLOS Guidance Law Design Using Ant Colony Optimization

Hadi Nobahari and Seid H. Pourtakdoust

Sharif University of Technology, P.O. Box: 11365-8639, Tehran, Iran
nobahari@mehr.sharif.edu
pourtak@sharif.edu

Abstract. The well-known ant colony optimization meta-heuristic is applied to design a new command to line-of-sight guidance law. In this regard, the lately developed continuous ant colony system is used to optimize the parameters of a pre-constructed fuzzy sliding mode controller. The performance of the resulting guidance law is evaluated at different engagement scenarios.

1 Introduction

The principle of Command to Line-of-Sight (CLOS) guidance law is to force the missile to fly as nearly as possible along the instantaneous line joining the ground tracker and the target, called the Line-of-Sight (LOS) [1,2,3,4,5,6,7,8,9]. Theoretically, the missile-target dynamic equations are nonlinear and time-varying, partly because the equations of motion are described in an inertial frame, while aerodynamic forces and moments are represented in the missile and target body frames. Many different control techniques have been used to design different CLOS guidance laws, examples of which are optimal control theory [3,6], feedback linearization [4], polynomial method [5], supervisory control [9], and so on. However, these methods have resulted in rather complicated controllers, and some of them require the knowledge of the maneuvering model of the target.

In recent years CLOS guidance laws based on Fuzzy Logic Control (FLC) have been presented [7,8]. Fuzzy logic was proposed by Professor Lotfi Zadeh in 1965, at first as a way of processing data by allowing partial set membership rather than crisp membership. Soon after, it was proven to be an excellent choice for many control system applications since it mimics human control logic. FLC can model the qualitative aspects of human knowledge and reasoning processes without employing precise quantitative analyses. It also possesses several advantages such as robustness and being a model-free, universal approximation and a rule-based algorithm. However, the stability analysis for general FLC systems is still lacking. To cope with this deficiency, a combination of FLC and the well-known Sliding Mode Control (SMC) has been proposed in recent years, called Fuzzy Sliding Mode Control (FSMC) [10,11,12]. The stability of FSMC can be proved in the Lyapunov sense [12]. This technique has been widely used in many control applications, as well as the CLOS guidance problem [8]. The other advantage of the FSMC is that it has fewer rules than FLC. Moreover, by using

SMC, the system possesses more robustness against parameter variations and external disturbances.

There are some limitations to the development of fuzzy controllers, the most important of which is the knowledge does not always completely exist and the manual tuning of all the base parameters takes time. The lack of portability of the rule bases when the dimensions of the control system change, makes the later difficulty still more serious. To cope with these problems, the learning methods have been introduced. The first attempt was made by Procyk and Mamdani in 1979, with a "self tuning controller" [13]. The gradient descent method was used by Takagi and Sugeno in 1985 as a learning tool for fuzzy modeling and identification [14]. It was used by Nomura, et al. in 1991 as a self tuning method for fuzzy control [15].

The gradient descent method is appropriate for simple problems and real time learning, since it is fast. But it may be trapped into local minima. Also the calculation of the gradients depends on the shape of membership functions employed, the operators used for fuzzy inferences as well as the selected cost function. In 1998, Siarry and Guely used the well-known Genetic Algorithm to tune the parameters of a Takagi-Sugeno fuzzy rule base [16]. The same problem was solved by Nobahari and Pourtakdoust [17], using Continuous Ant Colony System (CACS) [18], which is an adaptation of the well-known Ant Colony Optimization (ACO) meta-heuristic to continuous optimization problems.

In this paper, CACS is used to optimize the parameters of a FSMC-CLOS guidance law. The optimization problem is to minimize the average tracking error obtained for 10 randomly generated engagement scenarios. In the simulation of these scenarios, the proposed random target maneuver in [3] is used. The performance of the optimal FSMC-CLOS, designed in this way, is then evaluated at some other engagement scenarios, considering both maneuvering and non-maneuvering targets. The simulation results show a good performance in both tracking dynamics and the final miss distance.

2 Problem Formulation

In this section the three-dimensional CLOS guidance is formulated as a non-linear time varying tracking problem. The three-dimensional pursuit situation is depicted in Fig. 1. The origin of the inertial frame is located at the ground tracker. The Z_I axis is vertical upward and the $X_I - Y_I$ plane is tangent to the Earth surface. The origin of the missile body frame is fixed at the missile center of mass with the X_I axis forward along the missile centerline.

Defining the LOS frame as depicted in Fig. 2, the three-dimensional guidance problem can be converted to a tracking problem. According to the definition of CLOS guidance law, reasonable choices for the tracking errors may be $\Delta\sigma = \sigma_t - \sigma_m$ and $\Delta\gamma = \gamma_t - \gamma_m$. The problem involves designing a controller to drive $[\Delta\sigma, \Delta\gamma]^T$ to zero. The same design algorithm will be applied for both azimuth and elevation angle control. In the following the azimuth angle control is chosen as an example.

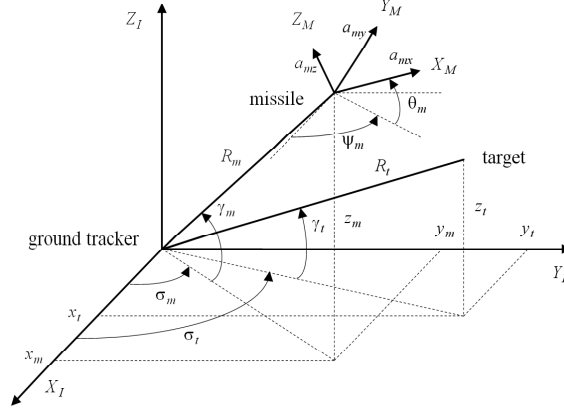


Fig. 1. Three-dimensional pursuit situation

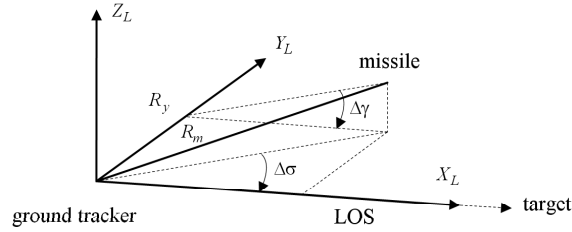


Fig. 2. Definition of the tracking error

Assume that $e(t) = \Delta\sigma$ represents the azimuth loop tracking error. The differential equation of the tracking error can be derived as [9]

$$\ddot{e} = f(\underline{e}, t) + (1/R_m(t))u(t) \quad (1)$$

Where $\underline{e} = [e(t), \dot{e}(t)]^T$ is the error state vector, $R_m(t)$ is missile to tracker range, $u(t) = -a_m(t)$ is the control variable, and

$$f(\underline{e}, t) = -(\ddot{R}_m(t)/R_m(t))e(t) - (2\dot{R}_m(t)/R_m(t))\dot{e}(t) + (1/R_m(t))a_t(t) \quad (2)$$

3 Sliding Mode Control

Let $s(\underline{e}) = 0$ denotes a hyper-surface in the space of the error state, which is called the sliding surface. The purpose of the sliding mode control is to force the error vector \underline{e} approach the sliding surface and then move along it to the origin. If the sliding surface is stable, the error \underline{e} will die out asymptotically. In this regard, let the sliding surface s be defined as follows

$$s(e, \dot{e}) = \left(\frac{d}{dt} + \lambda \right) e = \dot{e} + \lambda e \quad (3)$$

where λ is a positive constant. It is obvious from Eq. (3) that keeping the states of the system on the sliding surface will guarantee the tracking error vector \underline{e} asymptotically approach to zero. The corresponding sliding condition [19] is

$$\frac{1}{2} \frac{d}{dt} s^2 = s\dot{s} \leq 0 \quad (4)$$

The general control structure that satisfies the stability condition of the sliding motion, can be written as [19]

$$u = \hat{u} - K \operatorname{sgn}(s) \quad (5)$$

where \hat{u} is called the equivalent control law that is derived by setting $s = \dot{s} = 0$, and K is a positive constant. The sliding condition can be satisfied as long as K is chosen large enough [19].

3.1 Fuzzy Sliding Mode Control

As mentioned in section 1, the FSMC is a hybrid controller and inherits the advantages of both fuzzy and sliding mode controllers. The FSMC can be regarded as a fuzzy regulator that controls the variable s approach to zero. Let \mathbf{s} denote the fuzzy variable of the universe of discourse, s . Then some linguistic terms can be defined to describe the fuzzy variable \mathbf{s} , such as "zero", "positive large", "negative small", etc. Each linguistic term expresses a certain situation in the system. For example, \mathbf{s} is "zero" means that the state of system is on the sliding surface or is near to the sliding surface. Such linguistic expressions can be used to form fuzzy control rules as below

$$\begin{aligned} \text{Rule 1:} & \text{ If } \mathbf{s} \text{ is NB, then } \mathbf{u} \text{ is PB} \\ \text{Rule 2:} & \text{ If } \mathbf{s} \text{ is NM, then } \mathbf{u} \text{ is PM} \\ \text{Rule 3:} & \text{ If } \mathbf{s} \text{ is ZO, then } \mathbf{u} \text{ is ZO} \\ \text{Rule 4:} & \text{ If } \mathbf{s} \text{ is PM, then } \mathbf{u} \text{ is NM} \\ \text{Rule 5:} & \text{ If } \mathbf{s} \text{ is PB, then } \mathbf{u} \text{ is NB} \end{aligned} \quad (6)$$

where \mathbf{u} denotes the fuzzy variable of the control signal u , NB denotes "Negative Big", NM denotes "Negative Mid", ZO denotes "Zero", PM denotes "Positive Mid", and PB denotes "Positive Big". Each linguistic term is described by an associated membership function as shown in Fig. 3. In conventional fuzzy controller design, these membership functions are tuned by a trial-and-error procedure, based on certain physical sense or designer's experiences.

3.2 Definition of FSMC Design as an Optimization Problem

The FSMC considered here, involves a SISO fuzzy inference system with the variable \mathbf{s} as the input, and the variable \mathbf{u} as the output. The design problem is defined as finding the optimum values of the membership functions parameters. The more fuzzy terms are defined, the more fuzzy rules will be requested for

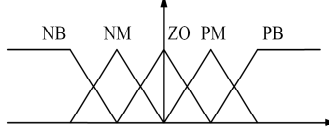


Fig. 3. Definition of fuzzy membership functions

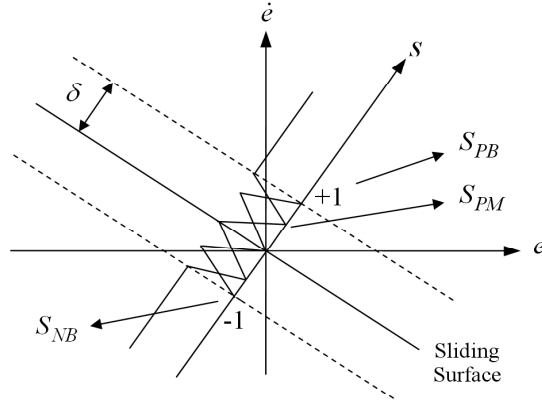


Fig. 4. Fuzzy sliding surface of a second order system

completeness. In this paper, five fuzzy terms are proposed for the measure of s , as follows

$$\mathbf{s} = \{\text{NB, NM, ZO, PM, PB}\} \quad (7)$$

One may choose more fuzzy terms if required. The associated five fuzzy terms for control energy are defined in a similar way as

$$\mathbf{u} = \{\text{NB, NM, ZO, PM, PB}\} \quad (8)$$

A scaling factor will be used to normalize the fuzzy set. The fuzzy sliding surface of a second order system is shown in Fig. 4, where δ represents the scaling factor. For normalized fuzzy sets, $S_{\text{NB}} = -1$, $S_{\text{PB}} = 1$ and $S_{\text{ZO}} = 0$ can be defined. The maximum control energy U_{max} is also bounded by physical limitations, that is $U_{\text{NB}} = -1$ and $U_{\text{PB}} = 1$ can also be defined. If the symmetry of fuzzy terms corresponding to \mathbf{s} and \mathbf{u} is assumed, the remaining design factors of the fuzzy system are the values of S_{PM} , U_{PM} , and the scaling factor δ . The optimal design problem of the FSMC then can be formulated as: to determine S_{PM} , U_{PM} , δ and the positive constant λ such that a given cost function is optimized.

4 Ant Colony Optimization

Ant algorithms were inspired by the observation of real ant colonies. An important and interesting behavior of ant colonies is their foraging behavior, and in

particular, how ants can find the shortest path without using visual cues. While walking from the food sources to the nest and vice versa, ants deposit on the ground a chemical substance called *pheromone* which makes a pheromone trail. Ants use pheromone trails as a medium to communicate with each other. They can smell pheromone and when they choose their way, they tend to choose paths with more pheromone. The pheromone trail allows the ants to find their way back to the food source or to the nest. Also, the other ants can use it to find the location of the food sources, which are previously found by their nest mates.

4.1 Ant Colony System

Ant Colony System (ACS) is one of the first discrete algorithms proposed based on ACO. At first it was applied to the well-known Traveling Salesman Problem (TSP) which is a discrete optimization problem. In this part we will shortly review the basic idea of ACS. Then in the subsequent part, the continuous version of ACS will be presented.

Ant Colony System uses a graph representation, like as the cities and the connections between them in TSP. In addition to the cost measure, each edge has also a desirability measure, called *pheromone intensity*. To solve the problem, each ant generates a complete tour by choosing the nodes according to a so called *pseudo-random-proportional state transition rule*, which has two major features. Ants prefer to move to the nodes, which are connected by the edges with a high amount of pheromone, while in some instances, their selection may be completely random. The first feature is called *exploitation* and the second one is a kind of *exploration*. While constructing a tour, ants also modify the amount of pheromone on the visited edges by applying a *local updating rule*. It concurrently simulates the *evaporation* of the previous pheromone and the *accumulation* of the new pheromone deposited by the ants while they are building their solutions. Once all the ants have completed their tours, the amount of pheromone is modified again, by applying a *global updating rule*. Again a part of pheromone evaporates and all edges that belong to the global best tour, receive additional pheromone conversely proportional to their length.

4.2 Continuous Ant Colony System

In this part, the lately developed Continuous Ant Colony System (CACS) is introduced. The interested readers can refer to [17,18] to find more details.

A continuous optimization problem is defined as finding the absolute minimum of a positive non-zero continuous cost function $f(x)$, within a given interval $[a, b]$, in which the minimum occurs at a point x_s . In general f can be a multi-variable function, defined on a subset of \mathbb{R}^n delimited by n intervals $[a_i, b_i]$, $i = 1, \dots, n$.

Continuous Ant Colony System (CACS) has all the major features of ACS, but certainly in a continuous frame. These are a pheromone distribution model, a state transition rule, and a pheromone updating rule. In the following these features are introduced.

Continuous Pheromone Model. Although pheromone distribution has been first modeled over discrete sets, like the edges of TSP, in the case of real ants, pheromone deposition occurs over a continuous space. The ants aggregation around the food source causes the most pheromone intensity to occur at the food source position. Then increasing the distance of a sample point from the food source will continuously decreases its pheromone intensity. CACS models the pheromone intensity, in the form of a normal Probability Distribution Function (PDF):

$$\tau(x) = e^{-\frac{(x - x_{\min})^2}{2\sigma^2}} \quad (9)$$

where x_{\min} is the best point found within the interval $[a, b]$ from the beginning of the trial and σ is an index of the ants aggregation around the current minimum.

State Transition Rule. In CACS, pheromone intensity is modeled using a normal PDF, the center of which is the last best global solution and its variance depends on the aggregation of the promising areas around the best one. So it contains exploitation behavior. In the other hand, a normal PDF permits all points of the search space to be chosen, either close to or far from the current solution. So it also contains exploration behavior. It means that ants can use a random generator with a normal PDF as the state transition rule to choose the next point to move to.

Pheromone Update. During each iteration, ants choose their destinations through the probabilistic strategy of Eq. (9). At the first iteration, there is no knowledge about the minimum point and the ants choose their destinations only by exploration. It means that they must use a high value of σ , associated with an arbitrary x_{\min} , to approximately model a uniform distribution function. During each iteration pheromone distribution over the search space will be updated using the acquired knowledge of the evaluated points by the ants. This process gradually increases the exploitation behavior of the algorithm, while its exploration behavior will decrease. Pheromone updating can be stated as follows: The value of objective function is evaluated for the new selected points by the ants. Then, the best point found from the beginning of the trial is assigned to x_{\min} . Also the value of σ is updated based on the evaluated points during the last iteration and the aggregation of those points around x_{\min} . To satisfy simultaneously the fitness and aggregation criteria, a concept of weighted variance is defined as follows:

$$\sigma^2 = \frac{\sum_{j=1}^k \frac{1}{f_j - f_{\min}} (x_j - x_{\min})^2}{\sum_{j=1}^k \frac{1}{f_j - f_{\min}}} \quad (10)$$

where k is the number of ants. This strategy means that the center of region discovered during the subsequent iteration is the last best point and the narrowness of its width depends on the aggregation of the other competitors around

the best one. The closer the better solutions get (during the last iteration) to the best one, the smaller σ is assigned to the next iteration.

During each iteration, the height of pheromone distribution function increases with respect to the previous iteration and its narrowness decreases. So this strategy concurrently simulates pheromone accumulation over the promising regions and pheromone evaporation from the others, which are the two major characteristics of ACS pheromone updating rule.

5 Numerical Results

In this section CACS is applied to optimize the parameters of a FSMC-CLOS guidance law, and the performance of the designed optimal guidance law is evaluated through different engagement scenarios. Ten different randomly generated engagement scenarios are used to evaluate each design point discovered by the ants. The cost function is defined as the average of the normalized tracking errors over the considered engagement scenarios. The normalized tracking error is defined as follows

$$r_n = \frac{1}{t_f} \int_0^{t_f} r(t) dt \quad (11)$$

where $r(t)$ is the distance between the missile and the LOS at time t . The average of the normalized tracking errors is defined as follows

$$y = \left(\frac{1}{10} (r_{n_1}^2 + r_{n_2}^2 + \dots + r_{n_{10}}^2) \right)^{\frac{1}{2}} \quad (12)$$

5.1 Mathematical Model of Missile and Target

The proposed equations of motion in [4] are used to simulate the behavior of missile and target. The acceleration limits of missile and target are $20(g)$ and $5(g)$, respectively. Other data used in simulations are the same as those in [4].

A random target maneuver similar to that proposed in [3], is utilized in simulations. It is assumed that target maneuvers about the LOS in a random fashion defined by RMS and bandwidth of the target acceleration. A stochastic representation of this maneuver will be generated by passing white noise, n_t , through a third order Butter-worth filter (Fig. 5). The values of K_t and ω_t , used in simulations, are $500(m/s^2)$ and $1(rad/s)$, respectively.

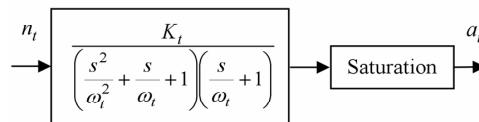


Fig. 5. Stochastic target maneuver model

5.2 Generation of the Random Engagement Scenarios

The cost function is defined based on the average performance obtained over 10 randomly generated engagement scenarios. These scenarios are pre-generated because the same situations are needed to evaluate different design points. In the generation of these primary scenarios, the following constraints are made

$$\begin{array}{ll}
 3000 \leq R_{0t} \leq 5000 & R_{0m} = 50 \text{ m} \\
 -180^\circ \leq \sigma_{0t} \leq 180^\circ & -5^\circ \leq \sigma_{0m} - \sigma_{0t} \leq 5^\circ \\
 20^\circ \leq \gamma_{0t} \leq 70^\circ & -5^\circ \leq \gamma_{0m} - \gamma_{0t} \leq 5^\circ \\
 300 \leq V_{0t} \leq 500 & V_{0m} = 150 \text{ m/s} \\
 -45^\circ \leq \psi_{0t} \leq 45^\circ & \psi_{0m} = \sigma_{0m} \\
 -20^\circ \leq \theta_{0t} \leq 20^\circ & \theta_{0m} = \gamma_{0m}
 \end{array}$$

In addition to the pre-generated initial conditions, for each scenario, two long lists of normalized zero-mean Gaussian random numbers are also generated and stored as the inputs of the target maneuver model, corresponding to a_{ty} and a_{tz} .

5.3 Optimization Results

The proposed CACS works based on the search and evaluation of different points in the solution space in a stochastic intelligent manner. The evaluation is done through the simulation of the missile-target engagement at the primary scenarios. The optimization problem can be defined as: find the values of S_{PM} , U_{PM} , δ , and λ such that the cost function, y , is minimized. The boundaries of the search space are defined as follows

$$0 < S_{PM} < 1, 0 < U_{PM} < 1, 0 < \delta < 1, 0 < \lambda < 10$$

Fig. 6 shows the history of y for different number of ants. The best results have obtained using 10 ants which is consistent with our previous results in

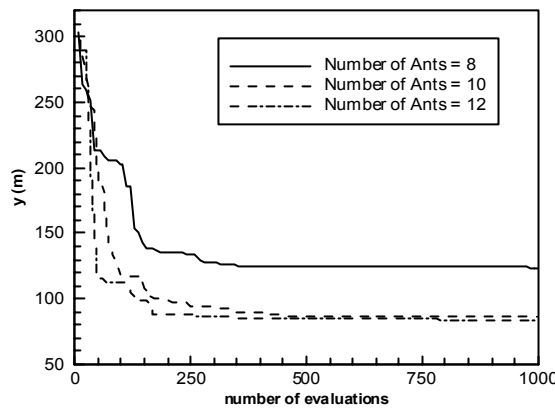


Fig. 6. History of the cost function as shown for different number of ants

[17,18]. The optimum values of the parameters obtained after 1000 evaluations are: $S_{PM} = 0.0074$, $U_{PM} = 0.9937$, $\delta = 0.3029$, and $\lambda = 2.74$.

5.4 Evaluation of the Optimal Design

The optimal set of parameters obtained based on the primary engagement scenarios, is again evaluated over two other scenarios. The two scenarios proposed in [4,8,9] are considered here. The first one represents a non-maneuvering target, while in the second one the target maneuvers with $a_{ty} = 5g$ and $a_{tz} = -g$ for the first 2.5 sec., and then $a_{ty} = -5g$ and $a_{tz} = 5g$ until interception. The initial condition data used to simulate these scenarios is given in table 1.

Figures 7 and 8 show the dynamic simulation results. It is clear that the new optimal FSMC-CLOS guidance law, designed using CACS, successfully drives the tracking error and as a result, the miss distance to zero. The obtained values

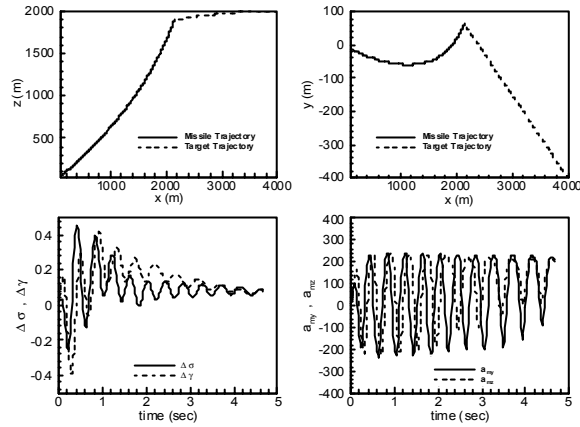


Fig. 7. Simulation results for the first scenario

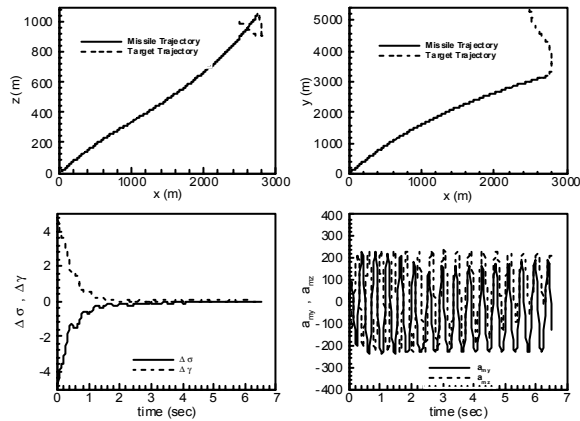


Fig. 8. Simulation results for the second scenario

Table 1. The initial condition used for different flight scenarios

| Parameter | Unit | Scenario 1 | Scenario 2 |
|--|------------|------------------|----------------------|
| $x_t(0), y_t(0), z_t(0)$ | <i>m</i> | 4000, -400, 2000 | 2500, 5361.9, 1000 |
| $\dot{x}_t(0), \dot{y}_t(0), \dot{z}_t(0)$ | <i>m/s</i> | -400, 100, 0 | 0, -340, 0 |
| $\psi_t(0), \theta_t(0)$ | <i>deg</i> | 165.96, 0 | -90, 0 |
| $x_m(0), y_m(0), z_m(0)$ | <i>m</i> | 100, -10, 50 | 14.32, 39.34, 3.36 |
| $\dot{x}_m(0), \dot{y}_m(0), \dot{z}_m(0)$ | <i>m/s</i> | 100, -10, 50 | 70.84, 151.92, 28.32 |
| $\psi_m(0), \theta_m(0)$ | <i>deg</i> | -5.71, 26.56 | 65, 9.59 |
| $\Delta\sigma(0), \Delta\gamma(0)$ | <i>deg</i> | 0, 0 | -5, 5 |

of miss distance for these two scenarios are 5.40 m and 3.96 m, respectively. The obtained results verify the ability of CACS to solve practical optimization problems such as guidance and control systems design.

6 Conclusion

In this paper the Continuous Ant Colony System (CACS), which is based on the well-known Ant Colony Optimization meta-heuristic was applied to design an optimal FSMC-CLOS guidance law. The optimization was done for different number of ants. The evaluation of each discovered point within the design space was done through the simulation of missile-target engagement at a number of randomly generated scenarios. The cost function was defined as the average of normalized tracking errors, corresponding to each scenario. Then the performance of the resulting optimal FSMC-CLOS guidance law was studied through some other new scenarios. Simulation results indicate a good performance for the new guidance law. This again shows the ability of CACS to solve practical optimization problems. The main advantage of CACS with respect to the other meta-heuristics such as Genetic Algorithm is its simplicity which is mainly due to its simple structure. CACS has only one control parameter which is the number of ants. This makes the parameter setting easier than many other optimization methods.

References

1. Garnell, P.: Guided Weapon Control Systems, 2nd Edition, Pergamon Press, Oxford, England, UK. (1980) chap. 7
2. Zarchan, P.: Tactical and Strategic Missile Guidance, 3rd Edition, AIAA Education Series. **176** (1997) 193–205
3. Kain, J. E., Yost, D. J.: Command to Line-of-Sight Guidance: A Stochastic Optimal Control problem. AIAA Guidance and Control Conference Proceedings (1976) 356–364
4. Ha, I. J., Chong, S.: Design of a CLOS Guidance Law via Feedback Linearization. IEEE Transactions on Aerospace and Electronic Systems **28** (1) (1992) 51–62

5. Parkes, N. E., Roberts, A. P.: Application of Polynomial Methods to Design of Controllers for CLOS Guidance. *IEEE Conference on Control Applications* **2** (1994) 1453–1458
6. Pourtakdoust, S. H., Nobahari, H.: Optimization of LOS Guidance for Surface-to-Air Missiles. *Iranian Aerospace Organization Conference* **2** (2000) 245–257
7. Arvan, M. R., Moshiri, B.: Optimal Fuzzy Controller Design for an Anti-Tank Missile. *International Conference on Intelligent and Cognitive Systems* (1996) 123–128
8. Lin, C. M., Hsu, C. F.: Guidance Law Design by Adaptive Fuzzy Sliding Mode Control. *Journal of Guidance, Control and Dynamics* **25** (2) (2002) 248–256
9. Nobahari, H., Alasty, A., Pourtakdoust, S. H.: Design of a Supervisory Controller for CLOS Guidance with Lead Angle. *AIAA Guidance, Navigation and Control Conference, AIAA-2005-6156* (2005)
10. Palm, R.: Robust Control by Fuzzy Sliding Mode. *Automatica* **30** (9) (1994) 1429–1437
11. Chen, C. L., Chang M. H.: Optimal Design of Fuzzy Sliding-Mode Control: A Comparative Study. *Fuzzy Sets and Systems* **93** (1998) 37–48
12. Choi, B. J., Kwak, S. W., Kim, B. K.: Design of a Single-Input Fuzzy Logic Controller and Its Properties. *Fuzzy Sets and Systems* **106** (3) (1999) 299–308
13. Procyk, T. J., Mamdani, E. H.: A linguistic self-organizing process controller. *Automatica, IFAC* **15** (1979) 15–30
14. Takagi, T., Sugeno, M.: Fuzzy identification of systems and its application to modeling and control. *IEEE Trans. Systems, Man and Cybernetics* **15** (1985) 116–132
15. Nomura, H., Hayashi, I., Wakami, N.: A Self Tuning Method of Fuzzy Control by Descent Method. *Proceedings of the International Fuzzy Systems Association, IFSA91, Bruxelles* (1991) 155–158
16. Siarry, P., Guely, F.: A Genetic Algorithm for Optimizing Takagi-Sugeno Fuzzy Rule Bases. *Fuzzy Sets and Systems* **99** (1998) 37–47
17. Nobahari, H., Pourtakdoust, S. H.: Optimization of Fuzzy Rule Bases Using Continuous Ant Colony System. *Proceeding of the First International Conference on Modeling, Simulation and Applied Optimization, Sharjah, U.A.E., ICMSAO-243* (2005) 1–6
18. Pourtakdoust, S. H., Nobahari, H.: An Extension of Ant Colony System to continuous optimization problems. *Lecture Notes in Computer Science* **3172** (2004) 294–301
19. Slotine, J. J. E., Li, W.: *Applied Nonlinear Control*, Prentice-Hall, Upper Saddle River, NJ (1991) chap. 7